Short Questions

Write the short answers of the following Questions:

Solve the following quadratic equations by factorization:

Q.1: $x^2 + 7x + 12 = 0$ (IIA-2017)

Sol. $x^2 + 7x + 12 = 0$

$x^2 + 4x + 3x + 12 = 0$

$x(x + 4) + 3(x + 4) = 0$

$(x + 4)(x + 3) = 0$

Either $x + 4 = 0$ OR $x + 3 = 0$

$x = -4$ | $x = -3$

S.S. $= \{-3, -4\}$

Q.2: $x^2 - x = 2$ (IIA-2019), (IA-2022)

Sol. $x^2 - x = 2$

$x^2 - x - 2 = 0$

$x^2 - 2x + x - 2 = 0$

$x(x - 2) + 1(x - 2) = 0$

$(x - 2)(x + 1) = 0$

Either $x - 2 = 0$ OR $x + 1 = 0$

$x = 2$ | $x = -1$

S.S. $= \{-1, 2\}$

Q.3: $x(x + 7) = (2x - 1)(x + 4)$ (IA-2019), (IIA-2021)

Sol. $x(x + 7) = (2x - 1)(x + 4)$

$x^2 + 7x = 2x^2 + 8x - x - 4$

$x^2 + 7x = 2x^2 + 7x - 4$

$x^2 + 7x - 2x^2 - 7x + 4 = 0$

$-x^2 + 4 = 0 \Rightarrow x^2 - 4 = 0$

$(x)^2 - (2)^2 = 0$

$(x - 2)(x + 2) = 0$

Either $x - 2 = 0$ OR $x + 2 = 0$

$x = 2$ | $x = -2$

S.S. $= \{-2, 2\}$

SOLUTION OF SHORT QUESTIONS

Q.4: $6x^2 - 5x = 4$ (IA-2018)

Sol. $6x^2 - 5x = 4$

$6x^2 - 5x - 4 = 0$

$6x^2 - 8x + 3x - 4 = 0$

$2x(3x - 4) + 1(3x - 4) = 0$

$(3x - 4)(2x + 1) = 0$

Either $3x - 4 = 0$ OR $2x + 1 = 0$

$3x = 4$ | $2x = -1$

$x = \frac{4}{3}$ | $x = \frac{-1}{2}$

S.S. $= \left\{-\frac{1}{2}, \frac{4}{3}\right\}$

Q.5: $3x^2 + 5x = 2$ (IIA-2016), (IA-2017), (IIA-2018), (IIA-2020), (IA-2021)

Sol. $3x^2 + 5x = 2$

$3x^2 + 5x - 2 = 0$

$3x^2 + 6x - x - 2 = 0$

$3x(x + 2) - 1(x + 2) = 0$

$(x + 2)(3x - 1) = 0$

Either $x + 2 = 0$ OR $3x - 1 = 0$

$x = -2$ | $3x = 1$

$x = \frac{1}{3}$

S.S. $= \left\{-2, \frac{1}{3}\right\}$

Q.6. $mx^2 + (1 + m)x + 1 = 0$

Sol. $mx^2 + (1 + m)x + 1 = 0$

$mx^2 + x + mx + 1 = 0$

$x(mx + 1) + 1(mx + 1) = 0$

$(mx + 1)(x + 1) = 0$

Either $mx + 1 = 0$ OR

$mx = -1$ | $x + 1 = 0$

$x = \frac{-1}{m}$ | $x = -1$

S.S. $= \left\{-1, -\frac{1}{m}\right\}$

Solve the following equations by completing the square:

Q.7: $x^2 - 2x - 899 = 0$ (IIA-2018)

SOLUTION OF SHORT QUESTIONS

Sol. $x^2 - 2x - 899 = 0$

$x^2 - 2x = 899$

Adding the square of one half of the coefficient of x i.e., $(1)^2$ on both sides

$x^2 - 2x + (1)^2 = 899 + (1)^2$

$(x - 1)^2 = 900$

$\sqrt{(x-1)^2} = \pm\sqrt{900}$

$x - 1 = \pm 30$

Either $x - 1 = 30$ OR $x - 1 = -30$

$x = 30 + 1$ | $x = -30 + 1$

$x = 31$ | $x = -29$

S.S. = $\boxed{\{-29, 31\}}$

Q.8: $2x^2 + 12x - 110 = 0$ **(IA-2018)**

Sol. $2x^2 + 12x - 110 = 0$

$2x^2 + 12x = 110$

$2(x^2 + 6x) = 110$

$x^2 + 6x = \frac{110}{2}$

$x^2 + 6x = 55$

Adding the square of one half of the coefficient of x i.e., $(3)^2$ on both sides

$x^2 + 6x + (3)^2 = 55 + (3)^2$

$(x + 3)^2 = 55 + 9$

$(x + 3)^2 = 64$

$\sqrt{(x+3)^2} = \pm\sqrt{64}$

$x + 3 = \pm 8$

$x = \pm 8 - 3$

Either OR

$x = 8 - 3$ | $x = -8 - 3$

$x = 5$ | $x = -11$

S.S. = $\boxed{\{-11, 5\}}$

Q.9: $x^2 + 5x - 6 = 0$ **(IIA-2016)**

Sol. $x^2 + 5x - 6 = 0$

$x^2 + 5x = 6$

SOLUTION OF SHORT QUESTIONS

Adding the square of one half of the coefficient of x i.e., $\left(\frac{5}{2}\right)^2$ on both sides

$$x^2+5x+\left(\frac{5}{2}\right)^2=6+\left(\frac{5}{2}\right)^2$$

$$\left(x+\frac{5}{2}\right)^2=6+\frac{25}{4}$$

$$\left(x+\frac{5}{2}\right)^2=\frac{24+25}{4}$$

$$\left(x+\frac{5}{2}\right)^2=\frac{49}{4}$$

$$\sqrt{\left(x+\frac{5}{2}\right)^2}=\pm\sqrt{\frac{49}{4}}$$

$$x+\frac{5}{2}=\pm\frac{7}{2}$$

$$x=\pm\frac{7}{2}-\frac{5}{2}$$

$$x=\frac{\pm7-5}{2}$$

Either

$$x=\frac{+7-5}{2}$$

$$x=\frac{2}{2}$$

$$x=1$$

OR

$$x=\frac{-7-5}{2}$$

$$x=\frac{-12}{2}$$

$$x=-6$$

S.S. = $\boxed{\{-6,1\}}$

Q.10: $x^2-6x+8=0$ (IA-2019)

Sol. Same as Q.2(i) of Ex# 1.1 (see page # 12)

SOLUTION OF SHORT QUESTIONS

Solve the following equations by quadratic Formula:

Q.11: $4x^2 + 7x - 1 = 0$

Sol. $4x^2 + 7x - 1 = 0$

Here $a = 4, \quad b = 7, \quad c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{(7)^2 - 4(4)(-1)}}{2(4)}$$

$$x = \frac{-7 \pm \sqrt{49 + 16}}{8}$$

$$x = \frac{-7 \pm \sqrt{65}}{8} \quad \Rightarrow \quad \text{S.S.} = \boxed{\left\{\frac{-7 \pm \sqrt{65}}{8}\right\}}$$

Q.12: $9x^2 - x - 8 = 0$ **(IA-2021)**

Sol. $9x^2 - x - 8 = 0$

Here : $a = 9$, $b = -1$, $c = -8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(9)(-8)}}{2(9)}$$

$$x = \frac{1 \pm \sqrt{1 + 288}}{18}$$

$$x = \frac{1 \pm \sqrt{289}}{18}$$

$$x = \frac{1 \pm 17}{18}$$

Either **OR**

Q.13: $x^2 - 3x - 18 = 0$ **(IIA-2020)**

Sol. $x^2 - 3x - 18 = 0$

Here : $a = 1$, $b = -3$, $c = -18$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-18)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9 + 72}}{2}$$

$$x = \frac{3 \pm \sqrt{81}}{2}$$

$$x = \frac{3 \pm 9}{2}$$

Either **OR**

SOLUTION OF SHORT QUESTIONS

$x = \frac{1+17}{18}$

$x = \frac{18}{18}$

$x = 1$

$x = \frac{1-17}{18}$

$x = \frac{-16}{18}$

$x = -\frac{8}{9}$

S.S. $= \boxed{\left\{-\frac{8}{9}, 1\right\}}$

$x = \frac{3+9}{2}$

$x = \frac{12}{2}$

$x = 6$

$x = \frac{3-9}{2}$

$x = \frac{-6}{2}$

$x = -3$

S.S. $= \boxed{\{-3, 6\}}$

Q.14: $x^2 - 3x = 2x - 6$

Sol. $x^2 - 3x = 2x - 6$

$x^2 - 3x - 2x + 6 = 0$

$x^2 - 5x + 6 = 0$

Here : $a = 1$, $b = -5$, $c = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 - 24}}{2}$$

$$x = \frac{5 \pm \sqrt{1}}{2}$$

$$x = \frac{5 \pm 1}{2}$$

Either **OR**

Q.15: $3x^2 - 5x - 2 = 0$

Sol. $3x^2 - 5x - 2 = 0$

Here : $a = 3$, $b = -5$, $c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-2)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{25 + 24}}{6}$$

$$x = \frac{5 \pm \sqrt{49}}{6}$$

$$x = \frac{5 \pm 7}{6}$$

Either **OR**

SOLUTION OF SHORT QUESTIONS

$x = \frac{5+1}{2}$ | $x = \frac{5-1}{2}$ | $x = \frac{5+7}{6}$ | $x = \frac{5-7}{6}$

$x = \frac{6}{2}$ | $x = \frac{4}{2}$ | $x = \frac{12}{6}$ | $x = \frac{-2}{6}$

$x = 3$ | $x = 2$ | $x = 2$ | $x = -\frac{1}{3}$

S.S. $= \{2, 3\}$ | S.S. $= \left\{-\frac{1}{3}, 2\right\}$

Discuss the Nature of the roots of the equations:

Q.16: $2x^2 - 7x + 3 = 0$

Sol. $2x^2 - 7x + 3 = 0$

Here : $a = 2, b = -7, c = 3$

$\text{Disc.} = b^2 - 4ac$

$= (-7)^2 - 4(2)(3)$

$= 49 - 24 = 25 = (5)^2$

Hence Roots are

Real, Rational and Unequal.

Q.17: $x^2 - 5x - 2 = 0$ (IA-2022)

Sol. $x^2 - 5x - 2 = 0$

Here : $a = 1, b = -5, c = -2$

$\text{Disc.} = b^2 - 4ac$

$= (-5)^2 - 4(1)(-2)$

$= 25 + 8 = 33$

Hence Roots are

Real, Irrational and Unequal.

Q.18: $x^2 + x + 1 = 0$ (IA-2017), (IA-2019), (IA-2021)

Sol. $x^2 + x + 1 = 0$

Here : $a = 1, b = 1, c = 1$

$\text{Disc.} = b^2 - 4ac$

$= (1)^2 - 4(1)(1)$

$= 1 - 4 = -3$

Hence roots are

Imaginary and Unequal.

Q.19: $x^2 - 2\sqrt{2}x + 2 = 0$ (IA-2016), (IA-2018), (IIA-2021)

Sol. $x^2 - 2\sqrt{2}x + 2 = 0$

Here : $a = 1, b = -2\sqrt{2}, c = 2$

$\text{Disc.} = b^2 - 4ac$

$= \left(-2\sqrt{2}\right)^2 - 4(1)(2)$

$= 4(2) - 8 = 8 - 8 = 0$

Hence roots are

Real, Rational and Equal.

SOLUTION OF SHORT QUESTIONS

Q.20: $9x^2 + 6x + 1 = 0$ (IIA-2016), (IIA-2018)

Sol. $9x^2 + 6x + 1 = 0$

Here : $a = 9$, $b = 6$, $c = 1$

Disc. $= b^2 - 4ac$

$= (6)^2 - 4(9)(1)$

$= 36 - 36 = 0$

Hence roots are **Equal and Real.**

For what value of 'k' the roots of the following equations are equal.

Q.21. $kx^2 + 4x + 3 = 0$ (IA-2019), (IIA-2020)

Sol. $kx^2 + 4x + 3 = 0$

Here $a = k$, $b = 4$, $c = 3$

As Roots are equal, So

Disc $= b^2 - 4ac = 0$

$\Rightarrow \quad (4)^2 - 4(k)(3) = 0$

$\Rightarrow \quad 16 - 12k = 0$

$\Rightarrow \quad -12k = -16$

$\Rightarrow \quad k = \dfrac{-16}{-12} \Rightarrow \boxed{k = \dfrac{4}{3}}$

Q.22. $2x^2 + 5x + k = 0$ (IIA-2016), (IIA-2019), (IA-2021) (IA-2022)

Sol. $2x^2 + 5x + k = 0$

Here $a = 2$, $b = 5$, $c = k$

As Roots are equal, So

Disc $= b^2 - 4ac = 0$

$\Rightarrow \quad (5)^2 - 4(2)(k) = 0$

$\Rightarrow \quad 25 - 8k = 0$

$\Rightarrow \quad -8k = -25$

$\Rightarrow k = \dfrac{-25}{-8} \Rightarrow \boxed{k = \dfrac{25}{8}}$

Q.23: Prove that the roots of the equation,

$(a+b)x^2 - ax - b = 0$ **are rational.** (IA-2016), (IIA-2021)

Sol. Same as Q.6(ii) of Ex# 1.2 (see page # 37)

Q.24: If the sum of the roots of $4x^2 + kx - 7 = 0$ **is 3. Find the value of 'k'.** (IA-2018), (IIA-2020)

Sol. Same as Q.2(iii) of Ex# 1.3 (see page # 41)

Q.25: Find the value of 'k' if the sum of the roots of equation

$(2k-1)x^2 + (4k-1)x + (k+3) = 0$ **is** $\dfrac{5}{2}$. (IIA-2018)

SOLUTION OF SHORT QUESTIONS

Sol. $(2k-1)x^2+(4k-1)x+(k+3)=0$

Here: $a = 2k-1$, $b = 4k-1$, $c = k+3$

As, Sum of Roots $= \frac{5}{2}$

$$-\frac{b}{a} = \frac{5}{2}$$

$$-\frac{4k-1}{2k-1} = \frac{5}{2}$$

$$-2(4k-1) = 5(2k-1)$$

$$-8k+2 = 10k-5$$

$$-8k+2 = 10k-5$$

$$-8k-10k = -5-2$$

$$-18k = -7$$

$$k = \frac{-7}{-18} \Rightarrow \boxed{k = \frac{7}{18}}$$

Find the sum and product of the roots of the following equations:

Q.26: $7x^2 - 5x + 4 = 0$

Sol. $7x^2 - 5x + 4 = 0$

Here $a = 7$, $b = -5$, $c = 4$

Sum of Roots $= -\frac{b}{a} = -\frac{(-5)}{7} = \boxed{\frac{5}{7}}$

Product of Roots $= \frac{c}{a} = \boxed{\frac{4}{7}}$

Q.27: $x^2 - 9 = 0$ **(IA-2017), (IIA-2017)**

Sol. Same as Q.1(ii) of Ex# 1.3 (see page # 40)

Q.28: $9x^2 + 6x + 1 = 0$ **(IA-2016), (IA-2019)**

Sol. $9x^2 + 6x + 1 = 0$

Here $a = 9$, $b = 6$, $c = 1$

Sum of Roots $= -\frac{b}{a} = -\frac{6}{9} = \boxed{-\frac{2}{3}}$

Product of Roots $= \frac{c}{a} = \boxed{\frac{1}{9}}$

Q.29: For what value of 'k' the sum of roots of equation $3x^2 + kx + 5 = 0$ may be equal to the product of roots? **(IA-2017), (IIA-2017), (IIA-2018)**

Sol. Same as Q.2(ii) of Ex# 1.3 (see page # 41)

SOLUTION OF SHORT QUESTIONS

Form the quadratic equations whose roots are:

Q. (i) $-2, -3$ (IA-2022)

Sol. $S = -2 + (-3)$ | $P = (-2)(-3)$

$S = -2 - 3 = -5$ | $P = 6$

$x^2 - Sx + P = 0$

$x^2 - (-5)x + 6 = 0 \Rightarrow \boxed{x^2 + 5x + 6 = 0}$

Q.31: $i\sqrt{3}, -i\sqrt{3}$ (IA-2016), (IA-2018), (IIA-2019)

Sol. $S = i\sqrt{3} + (-i\sqrt{3})$ | $P = (i\sqrt{3})(-i\sqrt{3})$

$S = i\sqrt{3} - i\sqrt{3}$ | $P = -(i)^2(\sqrt{3})^2$

$S = 0$ | $P = -(-1)(3) = 3 \quad \because i^2 = -1$

$x^2 - Sx + P = 0$

$x^2 - 0x + 3 = 0 \Rightarrow \boxed{x^2 + 3 = 0}$

Q.32: $-2 + \sqrt{3}, -2 - \sqrt{3}$ (IIA-2020)

Sol. $S = -2 + \sqrt{3} + (-2 - \sqrt{3})$ | $P = (-2 + \sqrt{3})(-2 - \sqrt{3})$

$S = -2 + \sqrt{3} - 2 - \sqrt{3}$ | $P = (-2)^2 - (\sqrt{3})^2$

$S = -4$ | $P = 4 - 3 = 1$

$x^2 - Sx + P = 0$

$x^2 - (-4)x + 1 = 0 \Rightarrow \boxed{x^2 + 4x + 1 = 0}$

If α, β are the roots of the equation $x^2 - 4x + 2 = 0$, find the equations whose roots are:

Q.33: $\frac{1}{\alpha}, \frac{1}{\beta}$

Sol. As α, β are the roots of the given equation.

$$x^2 - 4x + 2 = 0$$

Here $a = 1, \quad b = -4, \quad c = 2$

SOLUTION OF SHORT QUESTIONS

$$\text{Sum of Roots} = -\frac{b}{a} \qquad \text{Products of Roots} = \frac{c}{a}$$

$$\alpha+\beta = -\left(-\frac{4}{1}\right) = 4 \qquad \alpha\beta = \frac{2}{1} = 2$$

As, $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of the require equation.

$$S = \frac{1}{\alpha}+\frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} \qquad P = \left(\frac{1}{\alpha}\right)\left(\frac{1}{\beta}\right)$$

$$S = \frac{\alpha+\beta}{\alpha\beta} = \frac{4}{2} = \boxed{2} \qquad P = \frac{1}{\alpha\beta} = \boxed{\frac{1}{2}}$$

$$x^2 - Sx + P = 0 \Rightarrow x^2 - 2x + \frac{1}{2} = 0$$

Multiplying by 2, we get $\boxed{2x^2 - 4x + 1 = 0}$

Q.34: $-\alpha$, $-\beta$ **(IIA-2016)**

Sol. As α, β are the roots of the given equation.

$$x^2 - 4x + 2 = 0$$

Here $a = 1$, $b = -4$, $c = 2$

$$\text{Sum of Roots} = \frac{-b}{a} \qquad \text{Products of Roots} = \frac{c}{a}$$

$$\alpha+\beta = -\left(-\frac{4}{1}\right) = 4 \qquad \alpha\beta = \frac{2}{1} = 2$$

As, $-\alpha, -\beta$ are the roots of require equation.

$$S = -\alpha + (-\beta) \qquad P = (-\alpha)(-\beta)$$

$$S = -\alpha - \beta \qquad P = \alpha\beta$$

$$S = -(\alpha+\beta) = -4 \qquad P = 2$$

$$x^2 - Sx + P = 0$$

$$x^2 - (-4)x + 2 = 0 \Rightarrow \boxed{x^2 + 4x + 2 = 0}$$